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COMPARISON STUDY OF THE GEOMETRIC PARAMETER CALIBRATION METHODS

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Abstract

This paper presents a comparison study on the methods of geometric parameter calibration of robots. The study takes into consideration classical methods, which require measuring the position or location (position and orientation) of the terminal link using external sensors, and self calibration methods, which carry out the calibration of the geometric parameters using only the joint position sensors. This comparison focuses on the identifiable parameters, condition number of the observation matrix, and on the convergence rate of the solution. This study is carried out by simulating the Puma Stanford robots, but general conclusions are derived.

Key Words

Calibration, robots, static accuracy, identification, geometric parameters, identifiable parameters

1. Introduction

The absolute accuracy of a robot depends to a large extent on the accuracy of the numerical values of the geometric parameters used in the direct and inverse kinematic models. Geometric calibration of robots is the process by which the geometric parameters are precisely identified. In general, the geometric calibration is carried out by solving a system of linear or nonlinear equations, which are a function of the geometric parameters, the joint positions, and the location (position and orientation) of the end-effector frame.

A large variety of methods have been proposed for the calibration of the geometric parameters. They vary by the type of endpoint sensing, the endpoint constraints, and by the number of equations. In this paper the following methods have been used in the comparison:

- *Classical Open Loop Method:* this method is based on using a set of configurations for which the joint positions and the corresponding Cartesian coordinates of the terminal frame position or location are given [1–4].
- *Relative Location Measure:* the calibration is carried out using the joint positions and the relative Cartesian location between each couple of configurations.
- *Distance Measure:* this method uses the joint positions of a set of configurations and the distance between the terminal points of each couple of configurations.
- *Frame (or position) Link:* this autonomous method needs only the joint positions: no external sensor is needed. It can be applied for redundant robots or non-redundant robots, which can achieve the same location (or position in the case of position link) of the terminal frame by multiple configurations. The data used in the calibration correspond to the joint variables of some sets of robot configurations, giving the same Cartesian location (or position) of the terminal link of the robot [5–8].
- *Plane Link:* in this case, the calibration is carried out using the joint positions of a set of configuration whose terminal points are in the same plane [9–13].

This paper focuses primarily on the identifiable parameters of these methods and on their convergence rate. The geometric parameters are estimated using a linearized model, which is solved iteratively using a least squares criterion, and by updating the identified parameters and the observation matrix after each iteration. Simulations show that even in the case of large errors in the parameters, such techniques work very well.

The paper is organized as follows: the parameters defining the robot are presented in Section 2. Section 3 describes the calibration methods. Section 4 presents the simulation of the calibration methods on the Puma and Stanford robots as well as the comparison of the different methods. Section 5 is the conclusion.

2. Description of the Geometric Parameters

The geometric parameters are the constant parameters and the offsets of the joint positions that are used to calculate the location of the end-effector of the robot with respect to the fixed world frame.

We consider serial robots consisting of n joints and $n+1$ links. Link 0 is the base and link n is the terminal link, frame j is defined fixed on link j, we denote:

- frame -1: the fixed reference frame
- frame $n+1$: the end-effector frame

The end-effector location can be computed with respect to the reference frame by the direct geometric model:

$$^{-1}\mathbf{T}_{n+1} = {}^{-1}\mathbf{T}_0 . {}^{0}\mathbf{T}_n(q) . {}^{n}\mathbf{T}_{n+1} \quad (1)$$

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where ${}^{i}\mathbf{T}_j$ is the 4×4 transformation matrix defining frame j relative to frame i and q is the vector of joint variables.

The definition of the link frames used here follows the notations of the modified Denavit and Hartenberg method proposed by Khalil and Kleinfinger [14, 15]. Frame j is defined such that z_j is along the axis of joint j, and x_j is perpendicular to z_j and z_{j+1}. Frame j is defined relative to frame $j-1$ by the matrix ${}^{j-1}\mathbf{T}_j$, which is a function of the four parameters (α_j, d_j, θ_j and r_j) as shown in Fig. 1. The joint variable q_j is equal to θ_j if joint j is revolute and r_j if joint j is translational.

Figure 1. The geometric parameters of a serial robot.

Although frames ${}^{-1}\mathbf{T}_0$ and ${}^{n}\mathbf{T}_{n+1}$ can be arbitrarily defined, it has been shown that the calculation of ${}^{-1}\mathbf{T}_{n+1}$ can be obtained as a function of the geometric parameters of $n+2$ frames represented by four parameters for each frame (α_j, d_j, θ_j, and r_j) for $j = 0, \ldots, n+1$, except for the first frame for which $\alpha_0 = 0$, $d_0 = 0$ [15, 16].

In the calibration process, we have to identify the deviation of the real parameters from the nominal values, thus to identify: $\Delta\theta_0$, Δr_0 and $\Delta\alpha_j$, Δd_j, $\Delta\theta_j$, Δr_j (for $j = 1, \ldots, n+1$).

If the axis of joint j is parallel to the axis of joint $j-1$, an additional parameter $\Delta\beta_j$, which is called the Hayati parameter, must be considered [17]. The nominal value of β_j is equal to zero such that:

$${}^{j-1}\mathbf{T}_j =$$

$$\mathrm{Rot}(y, \beta_j).\mathrm{Trans}(x, d_j).\mathrm{Rot}(x, \alpha - j).\mathrm{Trans}(z, r_j).\mathrm{Rot}(z, \theta_j) \qquad (2)$$

Using the same procedure, one can also identify the errors on the joint angle gains giving the joint positions from the position sensor readings. They will be denoted by ΔK_j (for $j = 1, \ldots, n$) [4].

The geometric parameters of the Puma manipulator are given in Table 1 and those of the Stanford arm are listed in Table 2. We have supposed the same ${}^{-1}\mathbf{T}_0$ and ${}^{n}\mathbf{T}_{n+1}$ for both robots. Beside the parameters β_j, α_j, d_j, θ_j, and r_j we see the following parameters:

- σ_j: which defines the type of the frame

  $\sigma_j = 0$ if joint j is revolute

  $\sigma_j = 1$ if joint j is prismatic

  $\sigma_j = 2$ if frame j is fixed

- K_j: the joint angle gain of joint j , for $j = 1, \ldots, n$

Figure 2. Puma manipulator.

Figure 3. Stanford manipulator.

Table 1
The Geometric Parameters of the Puma (6R) Robot

j	σ_j	α_j	d_j	θ_j	r_j	β_j	K_j
0	2	0	0	$\pi/2$	0.5	0	0
1	0	0.1	0	θ_1	0	0	1
2	0	$-\pi/2$	0	θ_2	0.1491	0	1
3	0	0	0.4318	θ_3	0	0	1
4	0	$\pi/2$	-0.0203	θ_4	0.4331	0	1
5	0	$-\pi/2$	0	θ_5	0	0	1
6	0	$\pi/2$	0	θ_6	0.3^2	0	1
7	2	1.3	0.2	$\pi/2$	0.1	0	0

Units are given in meters for the distances and radians for the angles.

Table 2
The Geometric Parameters of the Stanford (RRPRRR) Robot

j	σ_j	α_j	d_j	θ_j	r_j	β_j	K_j
0	2	0	0	$\pi/2$	0.5	0	0
1	0	0.1	0	θ_1	0	0	1
2	0	$-\pi/2$	0	θ_2	0.2	0	1
3	1	$\pi/2$	0	0	r_3	0	1
4	0	0	0	θ_4	0	0	1
5	0	$-\pi/2$	0	θ_5	0	0	1
6	0	$\pi/2$	0	θ_6	0.3^1	0	1
7	2	1.3	0.2	$\pi/2$	0.1	0	0

Units are given in meters for the distances and radians for the angles.

[1] r_6 is not equal to zero because of the supposed terminal frame.

3. Calibration Methods

The calibration methods studied in this paper differ according to the variables used in the calibration model and the type of measuring device. A unified approach can be formulated using the framework suggested in [18]. According to this formulation, the kinematic calibration equation can be written in the general form:

$$0 = f(q, x, \eta_r) \qquad (3)$$

with

- x as the Cartesian variables giving the position and orientation of the terminal frame
- q as the $(n \times 1)$ joint variables vector
- η_r as the $(Np \times 1)$ vector of the real (unknown) values of the geometric parameters

Each method can be classified by a calibration index giving the number of equations of the function f [19].

The nonlinear calibration equation (3) can be linearized to get the differential equation:

$$\Delta y(q, x, \eta) = \Psi(q, \eta).\Delta\eta \tag{4}$$

with

- $\Delta\eta = \eta_r - \eta$, defining the vector of the errors on the geometric parameters
- η as the vector of the nominal values of the geometric parameters

To estimate $\Delta\eta$, (4) will be applied using a sufficient number of configurations $Q = q^1, \ldots q^m$ to obtain an over-constrained system of equations:

$$\Delta Y(Q, X, \eta) = \mathbf{W}(Q, \eta).\Delta\eta \tag{5}$$

with

$$\Delta Y = \begin{bmatrix} \Delta y^1(q^1, x^1, \eta) \\ \vdots \\ \Delta y^m(q^m, x^m, \eta) \end{bmatrix}, \mathbf{W} = \begin{bmatrix} \Psi(q^1, \eta) \\ \vdots \\ \Psi(q^m, \eta) \end{bmatrix}$$

where $\mathbf{W}$ is the $(r \times Np)$ observation matrix with $r \gg Np$, $X = x^1, \ldots, x^m$.

It can be seen that if some columns of $\mathbf{W}$ are dependent, and $\mathbf{W}$ is of rank $b < Np$, then relation (5) can be reduced to [20]:

$$\Delta Y(Q, X, \eta) = \mathbf{W_b}(Q, \eta).\Delta\eta_b \tag{6}$$

where $\mathbf{W_b}$ contains arbitrarily b independent columns of $\mathbf{W}$, the corresponding parameters are called the identifiable (or base) parameters. They will be denoted by the vector $\Delta\eta_b$.

The determination of the identifiable parameters $\Delta\eta_b$ must be done before the identification process. They can be obtained numerically using the QR decomposition of a matrix $\mathbf{W}$ similar to that defined in (5), but obtained using random configurations satisfying the constraints of the calibration method. The outline of this method is given in Appendix A.

Equation (6) will be solved to get the least squares error solution to the current parameter estimate. This procedure is iterated until the vector $\Delta\eta_b$ is sufficiently small. After each iteration, the geometric parameters will be updated in $\mathbf{W_b}$ and ΔY. In the following we will use the standard pseudo-inverse to estimate $\Delta\eta_b$. Other methods using statistical solutions are given in [18]. The condition number of $\mathbf{W_b}$ gives an index measure for the observability of the parameters in the calibration system of equations. To get good results the condition number must be close to one [16]. Other indices of the observability using the product of the singular values can be also used [21]. It has been shown that the condition number is more sensitive [22].

The elements of $\Delta\eta_b$ have different units: meters (for distances) radians (for angles) or even no units (gain coefficients). The effect of this heterogeneity can be reduced by a column normalization of the matrix $\mathbf{W_b}$ [19, 23]. Thus, (6) becomes:

$$\Delta Y = \sum_{j=1}^{b} \frac{w_j}{\|w_j\|}.\Delta\eta_j.\|w_j\| \tag{7}$$

where w_j is the jth column of the matrix $\mathbf{W_b}$.

It is to be noted that since the columns of $\mathbf{W_b}$ correspond only to the identifiable parameters, $w_j \neq 0$.

3.1 Classical Calibration Method

This method needs external sensors to measure the location of the terminal frame with respect to the world reference frame.

The nonlinear equation of the calibration is given as:

$$^{-1}\mathbf{T}_{n+1}(q, \eta_r) - {}^{-1}\mathbf{T}_{n+1}(x) = 0 \tag{8}$$

The linear differential model defining the deviation of the end-effector location due to the differential error in the geometric parameters can be obtained as [7, 16]:

$$\Delta x(q, \eta) = \mathbf{J}(q, \eta).\Delta\eta \tag{9}$$

with

- Δx represents the (6×1) vector of the position and orientation errors (representing the difference between measured and computed $^{-1}\mathbf{T}_{n+1}$)
- $\mathbf{J}$ is the $(6 \times Np)$ Jacobian matrix of frame $(n+1)$ with respect to the geometric parameters. Its columns can be calculated directly using the elements of the transformation matrices $^{-1}\mathbf{T}_j$ as given in Appendix B.

The calibration index of this method is equal to 6. If the external sensor gives only the position of the endpoint, only the first three equations of (9) will be used and the calibration index will be equal to 3.

3.2 Calibration Using Relative Location Measure

In this method we need an external sensor giving the relative location of the end-effector frame when moving from configuration q^a to q^b. Let the (4×4) matrix $^a\mathbf{F}_b$ denote this measure, thus:

$$[^{-1}\mathbf{T}_{n+1}(q^a, \eta_r)]^{-1}.{}^{-1}\mathbf{T}_{n+1}(q^b, \eta_r) = {}^a\mathbf{F}_b \tag{10}$$

The nonlinear calibration relation is given by:

$$^{-1}\mathbf{T}_{n+1}(q^b, \eta_r) - {}^{-1}\mathbf{T}_{n+1}(q^a, \eta_r).{}^a\mathbf{F}_b = 0 \tag{11}$$

The differential model of the first-order development is given as:

$$[\mathbf{J}(q^b, \eta) - \mathbf{J}2(q^a, \eta, {}^a\mathbf{F}_b)].\Delta\eta = \Delta x(q^a, q^b, \eta, {}^a\mathbf{F}_b \tag{12}$$

$\mathbf{J}(q^a, \eta)$ is obtained as given in Appendix B, while $\mathbf{J}2(q^a, \eta, {}^a\mathbf{F}_b$ can be obtained from the expressions of $\mathbf{J}(q^a, \eta)$ after replacing the vector $^{-1}\mathbf{P}_{n+1}(q^a, \eta)$ by the displacement vector of the transformation $^{-1}\mathbf{T}_{n+1}(q^a, \eta).{}^a\mathbf{F}_b$. The right-hand side of (12) is the differential error vector between the frames $^{-1}\mathbf{T}_{n+1}(q^b, \eta)$ and $^{-1}\mathbf{T}_{n+1}(q^a, \eta).{}^a\mathbf{F}_b$.

The calibration index of the method is equal to 6.

3.3 Calibration Using Distance Measure

This method handles the calibration of the geometric parameters of robots if the Cartesian distance of the endpoint can be measured using an external sensor when moving from configuration q^a to q^b.

Suppose two configurations, q^a and q^b, where the Cartesian distance between the position of their endpoints is Dr, the nonlinear calibration equation is given as:

$$[Px(q^b,\eta_r)-Px(q^a,\eta_r)]^2+[Py(q^b,\eta_r)-Py(q^a,\eta_r)]^2 \\ +[Pz(q^b,\eta_r)-Pz(q^a,\eta_r)]^2 = Dr^2 \quad (13)$$

The first-order differential model is given by:

$$\{2.[Px(q^b)-Px(q^a)].[Jx(q^b)-Jx(q^a)] \\ +2.[Py(q^b)-Py(q^a)].[Jy(q^b)-Jy(q^a)] \\ +2.[Pz(q^b)-Pz(q^a)].[Jz(q^b)-Jz(q^a)]\}\,\Delta\eta = Dr^2 - D^2 \quad (14)$$

where:

- D is the distance between the endpoints of configurations q^a and q^b, using the nominal parameters and the geometric model
- Jx, Jy, and Jz denote respectively, the first, second, and third row of the Jacobian matrix defined in (9).

The calibration index of this method is equal to 1.

3.4 Frame Link and Position Link Calibration Methods

The main problem of the previous methods is the need to have an accurate, fast, and inexpensive external sensor to measure the Cartesian variables. The frame link (or position link) method can be used for robots (redundant or not), where for a given location (or position) of the terminal effector, multiple configurations can be obtained from the inverse kinematic model. Let q^a and q^b represent two configurations giving the same location of the terminal link, then the nonlinear calibration model is given as:

$$^{-1}\mathbf{T}_{n+1}(q^a,\eta_r) = {}^{-1}\mathbf{T}_{n+1}(q^b,\eta_r) \quad (15)$$

The first-order differential model gives:

$$\left[\mathbf{J}(q^b,\eta)-\mathbf{J}(q^a,\eta)\right].\Delta\eta = \Delta x(q^a,q^b,\eta) \quad (16)$$

where $\mathbf{J}$ is the Jacobian matrix of frame $(n+1)$ with respect to the geometric parameter variations, as defined in (9).

Δx is the differential position and orientation vector between the tool locations $^{-1}\mathbf{T}_{n+1}(q^a,\eta)$ and $^{-1}\mathbf{T}_{n+1}(q^b,\eta)$.

The calculation of the identifiable parameters can be carried out, as demonstrated in Appendix A, by studying the QR decomposition of an observation matrix $\mathbf{W}$ calculated from (16) using a sufficient random couple of configurations, which give the same tool location. Such couple of configurations are obtained by supposing a random configuration q^a, then computing the terminal frame location $^{-1}\mathbf{T}_{n+1}(q^a)$. Finally q^b is obtained using the solution of the inverse geometric model for $^{-1}\mathbf{T}_{n+1}(q^a)$ and such that $q^b \neq q^a$.

In the case of the position link method, we take q^a and q^b giving the same terminal position such that $^{-1}\mathbf{P}_{n+1}(q^a,\eta_r) - {}^{-1}\mathbf{P}_{n+1}(q^b,\eta_r)$. Thus, we use only the first three equations of (16).

The calibration index, in the case of frame link, is equal to 6. It will be reduced to 3 in the case of position link.

3.5 Planar Calibration Methods

In this case the calibration will be carried out using the values of the joint positions of a set of configurations of the robot whose endpoints are in the same plane. Different methods based on this technique have been proposed previously [9–13].

Two methods are used in our study.

3.5.1 The First Method: Calibration Using Plane Equation

The general nonlinear equation of the calibration is:

$$a.Px(q,\eta_r)+b.Py(q,\eta_r)+c.Pz(q,\eta_r)+1=0 \quad (17)$$

where

a, b, and c represent the plane coefficients

Px, Py, and Pz represent the position coordinates of the terminal point in the world frame

Using a first-order development, the differential model is obtained as:

$$[Px(q,\eta)\;Py(q,\eta)\;Pz(q,\eta)\;a.Jx(q,\eta) \\ +b.Jy(q,\eta)+c.Jz(q,\eta)]\,.$$

$$\begin{bmatrix}\Delta a\\ \Delta b\\ \Delta c\\ \Delta\eta\end{bmatrix} = -a.Px(q,\eta)-b.Py(q,\eta)-c.Pz(q,\eta)-1 \quad (18)$$

where:

Jx, Jy, and Jz are respectively, the first, second, and third row of the Jacobian matrix defined in (9).

$Pu(q,\eta_r)$, for $u=x,y,z$ represent the u coordinate of the terminal point position at configuration q.

The coefficients of the plane are initialized by calculating the equation of the nearest plane to the terminal points of the given configurations.

If the coefficients of the plane are known, the parameters a, b, and c will be not identified. The corresponding columns and unknowns in (18) will be eliminated.

The calibration index of this method is equal to 1.

3.5.2 Second Method: Calibration using Normal Coordinates

In this method, we make use of the fact that the scalar product of the coordinates of the vector normal to the plane and of any vector between two points (i and j) in the plane is equal to zero [9, 12]. The main advantage of this procedure is that the coordinates of the normal can be obtained easily using inclinometers.

The system equation is thus:

$$a.\left\{Px(q^j,\eta_r) - Px(q^i,\eta)r)\right\} + b.\left\{Py(q^j,\eta_r) - Py(q^i,\eta_r)\right\} + c.\left\{Pz(q^j,\eta_r) - Pz(q^i,\eta_r)\right\} = 0 \quad (19)$$

Using a first-order development for η and assuming that the normal coordinates are known, we obtain:

$$\left\{a.\left[Jx(q^j) - Jx(q^i)\right] + b.\left[Jy(q^j) - Jy(q^i)\right] + c.\left[Jz(q^j) - Jz(q^i)\right]\right\}.\Delta\eta = -a.\left[Px(q^j) - Px(q^i)\right] - b.\left[Py(q^j) - Py(q^i)\right] - c.\left[Pz(q^j) - Pz(q^i)\right] \quad (20)$$

The calibration index of this method is equal to 1.

4. Evaluation of the Calibration Methods

In this section we study the previous methods on both the Puma and Stanford robots, and obtain some general conclusions.

The comparison focuses on: the identifiable parameters, the condition number of random data, the convergence to the real parameters, and the number of iterations to achieve convergence.

4.1 Identifiable Parameters

Tables 3 and 4 give the identifiable parameters of the two robots. The parameters indicated by '0' are not identifiable and they have no effect on the identification model. The parameters indicated by 'n' are not identified because they have been regrouped to some other parameters. The regrouped relations can be obtained as given in Appendix A. From Tables 3 and 4, the following general remarks are deduced:

1. The location measure method can identify the maximum number of parameters (36 parameters for a Puma robot and 34 for a Stanford robot), which corresponds to the following general relation, in agreement with that given in [24, 25]:
   Number of Identifiable Parameters $= 4(n+1) + 2 - 2n_P + n$.
   This number can be interpreted as:
   (a) 4 parameters for frames $1,\ldots,n+1$
   (b) 2 parameters for frame 0
   (c) $-2n_P$ because two parameters are not identifiable for each prismatic joint
   (d) n parameters of joint angle gains
2. The parameters of frames 0 and 1 have no effect on the model and cannot be identified when using the following methods: relative location measure, distance measure, point link and frame link.
3. Most of the parameters of frame n and $n+1$ are not identifiable in the frame link method.
4. Most of the parameters of frames 0, 1, and 7 are not identifiable in the planar methods. Some of them have been regrouped to other parameters.
5. The effect of the errors on the non-identifiable parameters will be calibrated through the parameters on which they have been regrouped.
6. The parameter β_j is not identifiable when $\alpha_j \neq 0$, that is to say, when the axis j is not parallel to the axis $j-1$.
7. The offsets of the joint variables $2,\ldots,n-1$ and all the gains K_j are identifiable in all the methods. q_n, however, is not identifiable in frame link method, whereas q_1 is not identifiable for the following methods: relative location measure, distance measure, point link, frame link, and unknown planar methods.
8. The parameter r_6 is not identifiable in the point link and plane link methods for the two robots; it represents the scale factor of those closed-loop methods. In the frame link method, the parameter r_6 has no effect and the scale factor is given by r_4 for the Puma manipulator and r_2 for the Stanford arm. It is to be noted that in the case of Stanford arm, the scale factor could be the prismatic variable r_3 (instead of r_6 or r_2) if we suppose that the joint gain K_3 is known and has not been identified.
9. The parameters α_7, θ_7, and r_7 are not identifiable in the case of the position measure method because the effector is just a point defined by r_6, θ_6, and d_7. For the same reason they are not identifiable for point link and plane link methods.

Table 3
Identifiable Parameters of the Puma Robot

Parameter	Position Measure	Location Measure	Relative Location Measure	Distance Measure	Point Link	Frame Link	Plane Link 1 (Known Plane)	Plane Link 1 (Unknown Plane)	Plane Link 2 (Known Plane)	Plane Link 2 (Unknown Plane)
$\theta 0$			0	0	0	0	n	n	n	n
$r0$			0	0	0	0		n	0	0
$\alpha 1$			0	0	0	0		n		n
$d1$			0	0	0	0	n	n	0	0
$\theta 1$			0	0	0	0		n		n
$r1$			0	0	0	0	n	n	0	0
$\beta 1$	n	n	0	0	0	0	n	n	n	n
$\alpha 2$										
$d2$										
$\theta 2$										
$r2$										
$\beta 2$	n	n	n	n	n	n	n	n	n	n
$\alpha 3$										
$d3$										
$\theta 3$										
$r3$	n	n	n	n	n	n	n	n	n	n
$\beta 3$										
$\alpha 4$										
$d4$										
$\theta 4$										
$r4$						n				
$\beta 4$	n	n	n	n	n	n	n	n	n	n
$\alpha 5$										
$d5$										
$\theta 5$										
$r5$										
$\beta 5$	n	n	n	n	n	n	n	n	n	n
$\alpha 6$										
$d6$										
$\theta 6$						0				
$r6$					n	0	n	n	n	n
$\beta 6$	n	n	n	n	n	0	n	n	n	n
$\alpha 7$	n			n	n	0	n	n	n	n
$d7$						0				

Parameter	Position Measure	Location Measure	Relative Location Measure	Distance Measure	Point Link	Frame Link	Plane Link 1 (Known Plane)	Plane Link 1 (Unknown Plane)	Plane Link 2 (Known Plane)	Plane Link 2 (Unknown Plane)
$\theta 7$	0			0	0	0	0	0	0	0
$r7$	n			n	n	0	n	n	n	n
$\beta 7$	n	n	n	n	n	0	n	n	n	n
$K1$										
$K2$										
$K3$										
$K4$										
$K5$										
$K6$										
Total	33	36	30	27	26	23	29	26	28	26 *

n: non-identifiable parameter, its effect is regrouped on some other parameters

0: non-identifiable parameter having no effect on the model

* in this method the coefficient c of the normal to the plane is not identifiable

Table 4
Identifiable Parameters of the Stanford Robot

Parameter	Position Measure	Location Measure	Relative Location Measure	Distance Measure	Point Link	Frame Link	Plane Link 1 (Known Plane)	Plane Link 1 (Unknown Plane)	Plane Link 2 (Known Plane)	Plane Link 2 (Unknown Plane)
$\theta 0$			0	0	0	0	n	n	n	n
$r0$			0	0	0	0		n	0	0
$\alpha 1$			0	0	0	0		n		n
$d1$			0	0	0	0	n	n	0	0
$\theta 1$			0	0	0	0		n		n
$r1$			0	0	0	0	n	n	0	0
$beta1$	n	n	0	0	0	0	n	n	n	n
$\alpha 2$										
$d2$										
$\theta 2$										
$r2$						n				
$\beta 2$	n	n	n	n	n	n	n	n	n	n
$\alpha 3$										
$d3$										
$\theta 3$	n	n	n	n	n	n	n	n	n	n
$r3$										
$\beta 3$	n	n	n	n	n	n	n	n	n	n
$\alpha 4$										
$d4$	n	n	n	n	n	n	n	n	n	n
$\theta 4$										
$r4$	n	n	n	n	n	n	n	n	n	n
$\beta 4$										
$\alpha 5$										
$d5$										
$\theta 5$										
$r5$										
$\beta 5$	n	n	n	n	n	n	n	n	n	n
$\alpha 6$										
$d6$										
$\theta 6$						0				
$r6$					n	0	n	n	n	n
$\beta 6$	n	n	n	n	n	0	n	n	n	n
$\alpha 7$	n			n	n	0	n	n	n	n
$d7$						0				

Parameter	Position Measure	Location Measure	Relative Location Measure	Distance Measure	Point Link	Frame Link	Plane Link 1 (Known Plane)	Plane Link 1 (Unknown Plane)	Plane Link 2 (Known Plane)	Plane Link 2 (Unknown Plane)
$\theta7$	0			0	0	0	0	0	0	0
$r7$	n			n	n	0	n	n	n	n
$\beta7$	n	n	n	n	n	0	n	n	n	n
$K1$										
$K2$										
$K3$										
$K4$										
$K5$										
$K6$										
Total	31	34	28	25	24	21	27	24	26	24 *

n: non-identifiable parameter, its effect is regrouped on some other parameters

0: non-identifiable parameter having no effect on the model

* in this method the coefficient c of the normal to the plane is not identifiable

4.2 Simulation Results

The following errors are supposed on the parameters of the Puma robot:

$$\begin{array}{llll} \Delta\alpha_2 : -0.05 & \Delta\alpha_5 : +0.05 & \Delta d_2 : +0.05 & \Delta d_3 : -0.05 \\ \Delta r_2 : -0.05 & \Delta r_5 : +0.05 & \Delta\theta_2 : +0.05 & \Delta\theta_4 : +0.05 \\ \Delta\theta_5 : -0.05 & \Delta K_1 : +0.05 & \Delta K_3 : -0.05 & \Delta K_6 : +0.05 \end{array}$$

The following errors are supposed on the parameters of the Stanford robot:

$$\begin{array}{llll} \Delta\alpha_2 : +0.05 & \Delta\alpha_5 : -0.05 & \Delta d_2 : +0.05 & \Delta d_3 : +0.05 \\ \Delta r_3 : +0.05 & \Delta r_5 : +0.05 & \Delta\theta_2 : -0.05 & \Delta\theta_4 : +0.05 \\ \Delta\theta_5 : -0.05 & \Delta K - 1 : +0.1 & \Delta K_3 : -0.1 & \Delta K_6 : -0.05 \end{array}$$

These errors give about 30 cm errors on the endpoint position for each robot. This error is very large with respect to the performance of industrial robots. Even in this extreme case, the standard pseudo-inverse works very well.

It is to be noted that these errors are identifiable in all the methods. The unknown vector $\Delta\eta$ contains all of the identifiable parameters of the corresponding method.

The calculation of the precision of the robots after the calibration is done with a set of 200 configurations, randomly distributed in the workspace of the robot, which are not used in the calibration process.

To get reliable results, each method has been tested using 10 different random sets of data. The number of configurations used for each method is taken so that we obtain 120 equations for the calibration, which is about four times the number of parameters.

4.3 Simulation without Noise

In this case we assumed exact measuring of q, and X and exact constraint links. The estimation process has been converged to the exact values of the geometric parameters in all the methods in about 5-8 iterations. The accuracy of the robot after calibration is seen to be 10^{-14} meters for the position and 10^{-14} radians for the orientation, which correspond to the numerical precision.

4.3.1 Simulation with Noise

In order to simulate a realistic calibration process, noisy data have been considered. The noise on the joint positions is randomly generated as equal to half a step of the encoders: we consider an encoder with 100,000 points for each joint, which corresponds to 6.6E-5 radians for a step. In the same way, we add normally zero mean distributed noise on the Cartesian measures and on all the virtual links in the autonomous methods with a variance of 0.1 millimeter on the position and 0.1 milliradians on the orientation.

It is to be noted that in the case of exact geometric parameters, the error on the position of the endpoint due to the imprecision on the joint position values is about 0.3 mm for the Puma robot and 0.2 mm for the Stanford robot.

Tables 5 and 6 give the identification results for the Puma and Stanford arms respectively. In all the methods, the convergence of the estimation of the geometric parameters has taken place within 6-13 iterations.

4.3.2 Remarks

From the foregoing simulations, the following remarks are given:

- In general, random data give good condition numbers for all methods, except plane link methods. Therefore, for planar points the user may need to select the location of the plane and the robot configurations by an optimization procedure.
- The normalization of the columns of the matrix Wb reduces its condition number.
- The condition number range is larger for the Stanford robot than for the Puma robot in the case of frame link. This can be explained by the fact that the inverse kinematic model of the Stanford arm has only four solutions, which limits the distribution of the configurations in the workspace.
- In general, the number of iterations increases when the condition number increases. This can be especially seen with the plane link methods, which have slower convergence than the other methods.
- In the case of joint angle noise, bias errors may result on the identified parameters, but the accuracy of the robot after calibration will be in the same order as the errors. For instance, in the case of the Puma robot, the joint noises give an error on the terminal point of about 0.3, which is in the same range as the errors due to the errors on the identified parameters after calibration in all the methods except in the case of some simulations concerning the plane link methods.
- We can observe that the final calibration accuracy of the robot using plane link identification methods with noisy data is less efficient than the other methods.
- When the level of noise is as given here, which is higher than typical values on industrial robots, the pseudo-inverse method works very well. If needed, statistical methods can take as initial conditions the solution obtained by the standard pseudo-inverse method.

5. Conclusion

This paper presents a comparison study on the geometric parameter calibration methods of robots. The study is carried out on the Puma and Stanford robots. Comparison of the methods focused on the identifiable parameters, the condition number, the convergence rate, and the precision. Noisy data have been also considered.

Table 5
Calibration of the Geometric Parameters of the Puma Robot with Noise

	Interval of maximum position error on the terminal point (m) After calibration	Interval of maximum orientation error on the terminal point (rad) After calibration	Convergence: Interval of number of interations	Interval of non-normalized condition number	Interval of normalized condition number
Position Measure	1.62E-04 : 3.19E-04	4.01E-04 : 5.90E-04	6 : 6	10.8 : 13.2	6.6 : 9.0
Location Measure	1.69E-04 : 2.74E-04	1.70E-04 : 2.64E-04	6 : 6	10.5 : 14.9	8.3 : 10.1
Relative Location Measure	1.08E-04 : 2.44E-04	1.33E-04 : 2.48E-04	6 : 6	9.9 : 12.3	6.9 : 10.7
Distance Measure	1.72E-04 : 3.22E-04	3.30E-04 : 6.03E-04	6 : 7	17.1 : 23.6	6.9 : 9.1
Point Link	1.26E-04 : 2.63E-04	3.19E-04 : 6.29E-04	6 : 6	9.9 : 12.5	6.1 : 8.0
Frame Link	6.89E-05 : 3.92E-04	1.23E-04 : 4.98E-04	6 : 6	9.6 : 16.4	6.9 : 10.7
Plane Link 1 (known plane)	2.61E-04 : 7.81E-04	4.48E-04 : 1.07E-03	6 : 7	12.5 : 197.1	8.7 : 31.3
Plane Link 1 (unknown plane)	2.56E-04 : 7.22E-04	4.29E-04 : 1.12E-03	7 : 9	18.6 : 472.4	9.4 : 37.9
Plane Link 2 (known normal)	3.14E-04 : 1.19E-03	5.79E-04 : 1.65E-03	6 : 7	14.2 : 41.7	9.0 : 27.2
Plane Link 2 (unknown normal)	2.73E-04 : 1.21E-03	4.99E-04 : 1.71E-03	7 : 13	32.4 : 4272.2	8.9 : 27.9

Table 6
Calibration of the Geometric Parameters of the Stanford Robot with Noise

	Interval of maximum position error on the terminal point (m) After calibration	Interval of maximum orientation error on the terminal point (rad) After calibration	Convergence: Interval of number of interations	Interval of non-normalized condition number	Interval of normalized condition number
Position Measure	1.69E-04 : 3.06E-04	3.60E-04 : 7.14E-04	6 : 7	11.0 : 14.0	6.6 : 8.2
Location Measure	1.46E-04 : 3.51E-04	1.27E-04 : 2.59E-04	6 : 6	14.0 : 20.6	10.6 : 16.6
Relative Location Measure	1.01E-04 : 2.52E-04	6.36E-05 : 1.82E-04	6 : 6	14.6 : 22.4	10.1 : 16.2
Distance Measure	1.79E-04 : 3.80E-04	3.25E-04 : 7.07E-04	6 : 7	19.0 : 26.1	9.6 : 13.7
Point Link	1.10E-04 : 2.14E-04	1.76E-04 : 5.74E-04	6 : 6	9.2 : 11.6	6.2 : 7.9
Frame Link	1.75E-04 : 7.82E-04	1.26E-04 : 3.63E-04	6 : 7	50.6 : 98.6	15.6 : 36.4
Plane Link 1 (known plane)	2.26E-04 : 6.26E-03	3.69E-04 : 6.66E-03	6 : 9	20.8 : 685.7	10.1 : 60.0
Plane Link 1 (unknown plane)	2.22E-04 : 1.85E-03	3.80E-04 : 2.16E-03	6 : 10	18.8 : 204.8	9.4 : 59.5
Plane Link 2 (known normal)	3.48E-04 : 1.94E-02	6.56E-04 : 1.73E-02	7 : 11	17.6 : 871.1	9.8 : 45.7
Plane Link 2 (unknown normal)	2.31E-04 : 1.60E-03	5.93E-04 : 1.54E-03	7 : 11	21.8 : 316.8	9.7 : 46.0

The identifiable parameters are determined for all of the methods using a numerical method based on QR decomposition, while the estimation of the geometric parameters is carried out using linearized iterative techniques. General conclusions have been obtained concerning the identifiable parameters of the different methods. It has been shown that the iterative-pseudo inverse method with column normalization works very well even in the case of noisy data and large initial errors in the geometric parameters.

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Appendix A: Calculation of the Identifiable Parameters

Let us consider the following overconstrained system of linear equations:

$$\Delta Y = \mathbf{W}.\Delta\eta \tag{21}$$

where $\mathbf{W}$ is $(r \times c)$ matrix with $r \gg c$. If b is the rank of $\mathbf{W}$, we can write:

$$\Delta Y = [\mathbf{W}_1\, \mathbf{W}_2].\begin{bmatrix} \Delta\eta_1 \\ \Delta\eta_2 \end{bmatrix} \tag{22}$$

where:

- $\mathbf{W}_1$ represents b independent columns of $\mathbf{W}$
- $\mathbf{W}_2$ represents the other $(c-b)$ columns of $\mathbf{W}$

We can write:

$$\mathbf{W}_2 = \mathbf{W}_1.\beta \tag{23}$$

with β as the $(b \times (c-b))$ matrix with constant elements.

Using (23), (22) can be written as:

$$\Delta Y = \mathbf{W}_1.\Delta\eta_b \tag{24}$$

with

$$\Delta\eta_b = \Delta\eta_1 + \beta.\Delta\eta_2 \tag{25}$$

The solution of the full rank system (24) will yield $\Delta\eta_b$, which is called the identifiable (or base) parameter vector. The matrix β is not needed in the identification process.

Numerically, the study of the identifiable parameters is equivalent to the study of the space spanned by the columns of the matrix $\mathbf{W}$.

The QR decomposition of the matrix $\mathbf{W}$ is given as [26, 27]:

$$\mathbf{Q}^T.\mathbf{W} = \begin{bmatrix} \mathbf{R} \\ \mathbf{0}_{(r-c)\times c} \end{bmatrix} \quad (26)$$

where:

$\mathbf{Q}$ is an $(r \times r)$ orthogonal matrix

$\mathbf{R}$ is a $(c \times c)$ upper triangular matrix

$\mathbf{0}_{i\times j}$ is the $(i \times j)$ matrix of zeros

Theoretically, the non-identifiable parameters are those whose corresponding elements on the diagonal of the matrix $\mathbf{R}$ are equal to zero. Assuming τ is the numerical zero, if $|R_{ii}|$ representing the absolute value of the (i, i) element of $\mathbf{R}$ is less than τ, the corresponding parameter $\Delta\eta_i$ is not identifiable. The numerical zero τ can be taken as [26]:

$$\tau = c.\epsilon.\max.|R_{ii}| \quad (27)$$

where ϵ is the machine precision.

To calculate the identifiable (base) parameters as a function of the standard parameters, let us permute the columns of $\mathbf{W}$ such that the first b columns are independent:

$$[\mathbf{W}.\mathbf{P}] = [\mathbf{W}_1\, \mathbf{W}_2] \quad (28)$$

where:

- $\mathbf{P}$ is a permutation matrix
- $\mathbf{W}_1$ represent b independent columns of $\mathbf{W}$
- $\mathbf{W}_2$ represent the other $(c - b)$ columns of $\mathbf{W}$.

The QR decomposition of $(\mathbf{W}.\mathbf{P})$ gives:

$$[\mathbf{W}_1\, \mathbf{W}_2] = [\mathbf{Q}_1\, \mathbf{Q}_2].\begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

where $\mathbf{R}_1$ is a $(b \times b)$ regular matrix. Then it becomes:

$$\mathbf{W}_2 = \mathbf{W}_1.\mathbf{R}_1^{-1}.\mathbf{R}_2 \quad (29)$$

Thus from (23) and (29) we get:

$$\beta = \mathbf{R}_1^{-1}.\mathbf{R}_2 \quad (30)$$

Appendix B: Calculation of the Jacobian Matrix

Assuming the (4×4) transformation matrix defining frame j relative to the fixed frame as:

$$^{-1}\mathbf{T}_j = \begin{bmatrix} s_j & n_j & a_j & P_j \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (31)$$

The calculation of the columns of the matrix $\mathbf{J}$ can be done as follows [4, 15, 16]:

$$J\alpha_j = \begin{bmatrix} s_{j-1} \times L_{j-1,n+1} \\ s_{j-1} \end{bmatrix} \quad Jd_j = \begin{bmatrix} s_{j-1} \\ o_{(3\times1)} \end{bmatrix}$$

$$J\theta_j = \begin{bmatrix} a_j \times L_{j,n+1} \\ a_j \end{bmatrix} \quad Jr_j = \begin{bmatrix} a_j \\ 0_{(3\times1)} \end{bmatrix} \quad (32)$$

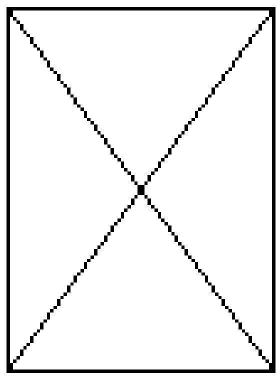
$$J\beta_j = \begin{bmatrix} n_{j-1} \times L_{j-1,n+1} \\ n_{j-1} \end{bmatrix} \quad Jk_j = Jq_j.\frac{\partial q}{\partial k_j}$$

where:

- $\times$ denotes the vector product
- $L_{j,n+1}$ is the (3×1) displacement vector between the origin of frame j and the origin of frame $n+1$ equal to $P_{n+1} - P_j$
- $0_{(3\times1)}$ is the (3×1) zero vector
- Jq_j is equal to $J\theta_j$ if joint j is revolute or Jr_j if joint j is prismatic

All the vectors of (32) refer to the measuring fixed frame.

Biographies



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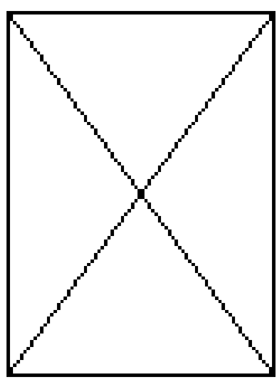

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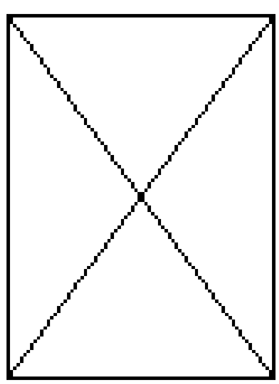

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